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MODELLING CONDITIONAL VOLATILITY
AND DOWNSIDE RISK
FOR ISTANBUL STOCK EXCHANGE

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Send correspondence to:
Amira Akl Ahmed
University of Benha, Egypt.
amira.ahmed@fcom.bu.edu.eg

Abstract

We investigated the impact of alternative variance equation specifications and different densities on the forecasting of one-day-ahead value-at-risk for the Istanbul stock market. The three employed models are symmetric GARCH(1,1) of Bollerslev (1986), symmetric GARCH(1,1) of Taylor (1986) and APGARCH(1,1) of Ding et al. (1996) models, under three distributions. The comparison focuses on two different aspects: the difference between symmetric and asymmetric GARCH (i.e., GARCH versus APGARCH) and the difference between normal-tailed and fat-tailed distributions (i.e., Normal, Student-t, and GED). The GARCH(1,1) of Taylor was found to be unjustified since convergence could not be achieved. Also, we examined if the estimated coefficients are time-varying. We executed a fixed size rolling sample estimation to provide the one-step-ahead variance forecasts required to generate the one-step-ahead VaR. Our results indicate that the APGARCH(1,1) with t-distribution model outperform its competitors regarding out-of-sample forecasting ability. Moreover, we found that the power transformation parameter of APGARCH model was time-variant. In contrast, degrees of freedom of t-distribution and thickness parameter of GED distribution are time-invariant indicating that fat-tailedness of innovation does not change over time. Thus, these findings suggest that the student-t APGARCH(1,1) model could be used by conservative investors to evaluate their investment risk. Also, both exchanges and regulators may benefit from using that model when the market faces turmoil and becomes more volatile.

JEL Classifications: C32, C52, C53, G15.

Keywords: value-at-risk, fat-tails, leverage effect, rolling sample, financial markets.

ملخص

نقوم بالتحقيق في تأثير مواصفات التباين المعادلة البديلة وكثافات مختلفة على التنبؤ يوما متقدما بنقطة واحدة القيمة المعرضة للخطر لسوق الأوراق المالية اسطنبول. نماذج العاملات الثلاث متماثل جارش (1،1) من بورسلف (1986)، متماثل جارش (1،1) من تايلور (1986) ومتماثل ابجارش (1،1) من دينغ وآخرون (1996). نماذج، تحت ثلاثة التوزيعات. وتركز المقارنة على جانبين مختلفين: الفرق بين متماثل جارش المتماثلة وغير المتماثلة (أي متماثل جارش مقابل متماثل ابجارش) والفرق بين التوزيع الطبيعي الذيل والذيل الدهون. تم العثور على متماثل جارش (1،1) من تايلور أن يكون غير مبرر منذ التقارب لا يمكن أن يتحقق. أيضا، درسنا إذا كانت المعاملات المقدرة هي زمنية متفاوتة. نفذنا حجم ثابت عينة المتداول تقدير لتقديم توقعات التباين خطوة للأمام واحدة اللازمة لتوليد القيمة المعرضة للخطر، خطوة متقدما بنقطة واحدة. نتائجنا تشير إلى أن ومتماثل ابجارش (1،1) مع نموذج توزيع تفوق منافسيها فيما يتعلق بمدى قدرة التنبؤ خارج العينة. وعلاوة على ذلك، وجدنا أن المعلمة تحول قوة النموذج ومتماثل ابجارش كان الوقت البديل. في المقابل، درجات الحرية لتوزيع تى للزمن .

1. Introduction

The second moment of returns has been commonly accepted as a proxy measure for risk [(Markowitz (1959); Sharpe (1964); Lintner (1965)]. Thus, conditional variance plays a crucial role in asset pricing, the portfolio selection, and risk management. Immense efforts have been devoted to modelling and forecasting stock market volatility using the family of ARCH models due to their ability to capture the dynamics of the conditional variance (e.g. volatility clustering and persistence) [Bollerslev et al. (1992); Ser-Huang and Granger (2003)]. The presence of volatility clustering is not distinctive to the squared returns of an asset's price since; in general, the absolute changes in an asset's price also exhibit volatility clustering. The common use of a squared term in this role is a reflection of the normality assumption traditionally invoked regarding the data[1]. Subsequent research, trying to improve capturing the stylized characteristics of financial series, gave birth to two competing families of GARCH models. Given that absolute returns exhibit strong long-term dependency, Taylor (1986) introduced a competitive family by specifying a power term of unity and directly linking the conditional standard deviation of a series to lagged absolute residuals and past standard deviations. Both models of Bollerslev (1986) and Taylor (1986) assume that conditional volatility is affected symmetrically by positive and negative innovations ignoring the leverage effect[2]. Equity returns are found to be negatively correlated with changes in returns volatility. In other words, volatility tends to fall in response to good news (i.e., excess returns higher than expected) and to rise in response to bad news (i.e., excess return lower than predicted).

The inclusion of any power term acts to capture volatility clustering. The preference given to squared terms (Bollerslev, 1986) or even a power of unity (Taylor, 1986) is inherited from the Gaussian assumption traditionally made in many modelling exercises. This could be investigated as, in this context, the expected square returns could be directly related to the variance and also the expected absolute returns to the standard deviation (Ane′, 2006). However, high-frequency financial time series are very likely to exhibit non-normal error distributions. Thus, squaring the returns or taking their absolute values imposes a structure on non-normal time series data that may furnish sub-optimal modelling and forecasting performance relative to other power terms [Brooks et al. (2000); Ane′ (2005)]. For this reason, Ding et al. (1993) suggested a new class of models known as the Asymmetric Power GARCH (APGARCH) wherein the power term by which the data are transformed, is estimated within the model instead of being imposed by the researcher. Thus, this model permits an infinite range of transformations inclusive of any positive value. It has been demonstrated to comprise a variety of ARCH specifications [Ding et al.(1993); Tse and Tsui (1997)].

The need for modelling the downside risk has rapidly grown in response to the financial crisis (i.e. the market crash in 1987, the Asian crisis in 1997, and the recent American subprime mortgage crisis). Thus, modelling the Value-at-Risk (VaR), serving as a risk management tool, has become one of the key measures of financial market risk. ARCH models have been extensively applied in the estimation of VaR for exchange rates, stock, and commodity markets [Füss et.al. (2007); Füss et al. (2010); Iqbal et al. (2010); Hung et al. (2008); Bams et al. (2005); Ane′ (2006)]. Füss et al. (2007) and Füss et al. (2010) concluded that GARCH-type VaR outperform other VaR approaches [i.e. conventional VaR, and the Cornish Fisher-VaR]. There is no conclusive evidence regarding the superiority of APGARCH model in modelling financial time series and forecasting VaR [Giot and Laurent (2003a, 2003b); Huang and Lin (2004);

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[1] If a data series follows the normal distribution, then we can characterize its distribution completely by its first two moments. Thus, it is appropriate to focus on a squared term and hence a measure of the volatility.

[2] GARCH models have some drawbacks in asset pricing applications. GARCH models impose non-negativity restrictions to ensure positivity of conditional variance which may unduly restrict the dynamics of the conditional variance process. Another drawback of GARCH modelling concerns the interpretation of the persistence of shocks to conditional variance because the usual norms measuring persistence often do not agree. The issue of volatility persistence is central in assets pricing. If shocks to volatility persist indefinitely, they are likely to move the whole term structure of risk premia. Accordingly, they are likely to significantly influence investment on long-lived capital goods (Nelson, 1991).

Tse and Tsui (1997); Mckenzie and Mitchell (2002); Ane′ (2005); Brooks et al. (2000)]. Abad et al. (2013) conducted a comprehensive review of value at risk methodologies. They concluded that the empirical evidence of the distribution performance in estimating VaR is **not** inclusive. Thus, the best distribution and the best GARCH model to predict VaR are still practical issues.

To enhance the role of its stock market and attract foreign investors, Turkey has taken significant actions to liberalize and improve the trading environment[3]. FTSE has recently classified ISE as developed exchange since it has passed the transparency criteria and other criteria of market quality set by FTSE (Ahmed, 2011; 2014). Due to the convenient investment environment, the share of foreign investors in the ISE reached 70% of market capitalisation in 2007 (Kasman and Torun, 2007). Compared to other exchanges in the Middle East and North Africa (MENA) region[4], the ISE is considered the most liquid during period 1995-2009 (Ahmed, 2011; 2014). In 2007, the ISE was ranked to be the tenth best-performing exchange among the WFE[5] members and the fifth best-performing exchange in Europe region (Kasman and Torun, 2007). The ISE is integrated with international financial markets. Ergun and Nor (2010) found significant volatility spillovers from the United States (NASDAQ) to the ISE for the period 1998-2008. Moreover, Celikkol et al. (2010) confirmed that the variance of ISE has significantly increased after the bankruptcy of Lehman Brothers. Ozun et al. (2010) used the time series of daily returns of the Istanbul Stock Index-100 (ISE-100) from January 2nd, 2002 to April 18th, 2007 and concluded that filtered extreme value theory are superior to the parametric VaR models regarding capturing fat-tails in stock returns than parametric VaR models.

Due to its role in portfolio diversification and risk management purposes, modelling volatility[6] of the ISE for the most recent period is of crucial importance for international and domestic investors given, the fluctuations in the volume of international portfolio investments and the unstable political environment in the MENA region. The current paper aims at examining the impact of alternative variance models under different distribution on forecasting the one-day-ahead VaR for ISE during the most recent period. Further, we contribute to the literature by examining if the estimated parameters change across time. We address two questions: do the estimated parameters of employed GARCH model change across time? Do fat-tails and asymmetry matter in forecasting one-day-ahead VaR for the ISE? We hypothesize that (1) the time-varying parameters reflect the fact that structural properties and trading behaviour alter over time (Xekalaki and Degiannakis, 2010), and (2) asymmetric GARCH models with fat tails are expected to perform better than symmetric ones with normally distributed innovations in forecasting one-day-ahead VaR.

To investigate those issues, the current paper applies APGARCH(1,1) and GARCH(1,1) models under three conditional densities (i.e. normal, Student t, and generalised error distributions) onto the MSCI-Turkey equity index for the period starting from 2nd January 2006 to 28th August 2015. Employed Models are evaluated according to their ability to generate the most accurate one-day-ahead VaR at 95% level of confidence. In order to compute the one-step-head variance forecasts required to create the one-step-ahead VaR, We use a fixed size

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[3] Actions taken by Turkey include the removal of restriction imposed on access of foreign investors to capital markets since 1989. Also, the introduction of American depository receipts in 1990 besides adopting automated trading systems in 1993. Additionally, regulatory reforms that include the establishment of regulatory bodies to ensure shareholders' protection and to monitor market activities. To achieve international comparability in accounting disclosure, Turkey has amended its national accounting standards to converge with the international set of financial reporting and accounting standards (Ahmed, 2011; 2014).

[4] These exchanges are those of Cairo, Tel Aviv, Amman and Casablanca.

[5] EFE stands for World Federation of Exchanges.

[6] In the 1950s, Markowitz established the mean-variance theory of combining risky assets to minimize the volatility of portfolio at any preferred mean return. The position of optimum mean-variance combinations is called the efficient frontier, on which all rational investors want to be located. The full risk of a well-constructed and diversified portfolio should be less than the sum of the risks in the portfolio's individual components.

rolling window of 1800 to produce 720 one-step- ahead volatility projections. With the aim of achieving closer real world accuracy, Xekalaki and Degiannakis (2010) recommended re-estimating models every trading day. Thus, we use the first 1800 observations to estimate the parameters of the two models under the three employed distributions. The estimation window is then moved forward by one day, and the model of concern is re-estimated. This process keeps running forward day by day until the end of the entire data set. Employing the rolling window procedure to re-estimate the model parameters each trading day will allow testing if the estimated parameters of the variance equation are time-varying.

The rest of paper is organised in the following way. Section 2 presents the volatility and VaR models employed in the paper. Then, the data description, preliminary analysis, and empirical findings (including the estimation of volatility models, VaR calculations and the out-of- sample forecasting results) are going to be reported in Section 3. Finally, section 4 concludes.

2. Methodology

2.1 Dynamics of conditional volatility

Assume that the adjusted closing price of a market index at time t is denoted by P_t . The continuously compounded returns R_t at time t, is computed as shown in Eq.1. If the stock return series displays significant first order autocorrelation, an AR(1) specification is employed as shown in Eq.2., where ε_t is the error term in period t. If the t-statistic associated with the autoregressive parameter μ_1 in Eq.2 is found to be insignificant, then a and näve no-change mean equation, expressed in Eq.3, is specified (Brooks et al, 2000)..

$$R_t = 100 * \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

$$R_t = \mu_0 + \mu_1 R_{t-1} + \varepsilon_t \tag{2}$$

$$R_t = \mu_0 + \varepsilon_t \tag{3}$$

The error term in the mean equation, whether Eq. 2 or 3 is specified, could be decomposed as shown in Eq.4, where e_t in the current study is assumed to follow normal [i.e. $e_t \sim N(0,1)$] , or Student t with v degrees of freedom greater than two $[e_t \sim t(0,1,v)]$, or generalised error distribution with ξ thickness of tail parameter that expected to be less than two for leptokurtoic distribution $[e_t \sim GED(0,1,\xi)]$

$$\varepsilon_t = \sigma_t e_t \tag{4}$$

The asymmetric power GARCH(1,1) [henceforward APGARCH(1,1)] proposed by Ding et al. (1993) specifies σ_t as expressed in Eq. 5.

$$\sigma_t^\delta = \omega_0 + \omega_1(|\varepsilon_{t-1}| - \gamma_1 \varepsilon_{t-1})^\delta + \beta_1 \sigma_{t-1}^\delta \tag{5}$$

$$\omega_0 \geq 0, \ \omega_1 > 0 , \ \ < \gamma_1 < 1 , \beta_1 \geq 0 \ \ \delta > 0$$

Where ω_1 and β_1 represent the standard ARCH and GARCH parameters respectively, γ_1 represents the leverage parameter, and δ corresponds to the optimal power that plays the role of a Box-Cox transformation of σ_t. A positive (negative) value of γ_1 means that past negative (positive) shocks have a deeper impact on current conditional volatility than past positive shocks (Haung and Lin, 2004). A significant advantage of the APGARCH structure is its flexibility to nest both conditional variance and conditional standard deviations of both Bollerslev (1986) and Taylor (1986), respectively (Ane´, 2005).

The abovementioned APGARCH(1,1) model nests some other ARCH specifications as special cases. For example, if δ=2 and $\gamma_1 = 0$, then we obtain the standard GARCH(1,1) of Bollerslev

(1986)[7]. The standard GARCH(1,1), expressed in Eq. 6, model the second moment as a function of lagged squared residuals (representing news about volatility from the previous period) and past variance (representing the effect of old news on volatility). The persistence of shocks to volatility depends on the sum of ARCH and GARCH coefficients (i.e., $\omega_1 + \beta_1$).

$$\sigma_t^2 = \omega_0 + \omega_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{6}$$

$$\omega_0 \geq 0, \ \omega_1 \geq 0, \ \beta_1 \geq 0 \text{ and } \ \omega_1 + \beta_1 < 0$$

Similarly, if $\delta = 1$ and $\gamma_1 = 0$, then we obtain Taylor's (1986) model, as shown in Eq.7, the model the conditional standard deviation to lagged absolute residuals and past standard deviation.

$$\sigma_t = \omega_0 + \omega_1 |\varepsilon_{t-1}| + \beta_1 \sigma_{t-1} \tag{7}$$

Parameters are estimated by employing the maximum likelihood procedure [i.e. Marquardt (1963)].

2.2 Value-at-Risk

The value at risk (VaR) could be defined as an amount of loss (measured in terms of £, $, €, etc.) on a portfolio or index with a given probability over a fixed number of days (Hung et.al, 2008). When measuring its risk, a portfolio can be considered as a multivariate system of individual returns or as a univariate return of the whole portfolio. In the current paper, we focus on the estimation of the VaR of a univariate series of returns. VaR has the remarkable property of expressing risk in only one figure and is the estimated loss of an asset, index, or portfolio, within a given period will only be exceeded by a certain small probability. Thus, the one day α % VaR shows the negative return that will not be exceeded within this day with a probability of $1 - \alpha$.

$$P[R_t < VaR_t \,|F_t] = \alpha \tag{8}$$

Where R_t is return series of an index or portfolio, F_t denotes the information set available at time t. The value at risk forecast for day t+1 conditional on the information set F_t can be thus expressed as in Eq. 9.

$$VaR_{t+1|t}(\alpha) = \mu_{t+1|t} + Z_\alpha \, \sigma_{t+1|t} \tag{9}$$

Where $\mu_{t+1|t}$ and $\sigma_{t+1|t}$ are the one-day-ahead mean and conditional standard deviation forecasts by the employed GARCH models and Z_α represents the left quantile of the employed distribution at α%. In the current study, all models are tested with 5% level of significance. The performance of VaR models (and implicitly of volatility models) will be assessed from two different perspectives (Ane′, 2006): accuracy and efficiency. The statistical adequacy will be tested based on backtesting measures of Kupiec (1995) and Christoffersen (1998) whereas the efficiency of different models will be compared via the binary and regulatory loss functions (Sarma et al. 2003).

2.2.1 Likelihood Ratio test for Unconditional Coverage: LR_{UC}

To assess the performance of competing models, we compute the failure rate, f, which is the proportion of times the realized returns R_t are below (i.e. more negative than) the forecasted VaR, $VaR_{t+1|t}(\alpha)$. If the VaR model is correctly specified, then f should be equal to the pre-specified VaR level α. The computation of f requires defining a sequence of ones (hits of the VaR) and zeros (no hits of the VaR). It is possible to test $H_0: f = \alpha$ against $H_0: f \neq \alpha$ by first observing that the number of hits follows a binomial distribution $T \sim (N, \alpha)$. The

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[7] Bollerslev (1986) extends the autoregressive conditional heteroscedasticity (ARCH) model of Engle (1982) to generalized ARCH (GARCH) where volatility is dependent on return shocks and past volatilities.

appropriate likelihood ratio test statistic of Kupiec (1995), known as the unconditional test and denoted as LR_{UC} , equals

$$LR_{UC} = 2 * ln\left[\left(1 - \frac{T}{N}\right)^{N-T} \left(\frac{T}{N}\right)^{T}\right] - 2 * ln[(1 - \alpha)^{N-T} (\alpha)^{T}] \tag{11}$$

Under the null hypothesis, the LR_{UC} statistic is asymptotically distributed as χ^2 (1). The test can be rejected a model for both too high or too low hit rates.

2.2.2 LR test for Conditional Coverage: LR_{CC}

The LR_{UC} is test is an unconditional test since it simply counts the number of hits over the entire period examined. The LR_{CC} test proposed by Christoffersen (1998) is used to test the conditional coverage. If the VaR estimates $VaR_{t+1|t}(\alpha)$ correctly incorporate the variance dynamics, the series of I_t must both exhibit correct unconditional coverage and serial independence. The LR_{CC} test is a joint test of these two properties. The LR_{CC} test statistic, which is asymptotically distributed as χ^2 (2) is defined by

$$LR_{UC} = 2 * ln[(1 - \pi_{01})^{n_{00}} (\pi_{01})^{n_{01}} (1 - \pi_{11})^{n_{10}} (\pi_{11})^{n_{11}}] - 2 * ln[(1 - \alpha)^{N-T} (\alpha)^{T}])$$

Where n_{ij} is the number of observation with the value i followed by j for i, j=0,1, while $\pi_{ij} = n_{ij}/\sum_j n_{ij}$ are the corresponding probabilities. The value i,j=1 indicates that a violation of the VaR level has occurred while i,j=0 denotes the opposite. The advantage of LR_{CC} test is that it can reject a model that can generates either too many or two few clustered hits.

2.2.3 Binary and Regulatory Loss Functions

The loss function evaluation method suggested by Lopez (1998) assign to each VaR estimate a numerical score that provides a measure of relative importance. The loss functions are defined with a negative orientation since they give higher scores when failures take place. VaR models are assessed by comparing the values of the loss function. A model which minimises the loss is preferred over the other models (Sarma et al., 2003). In the current study, we use two loss functions. The binary loss function (BLF) is a reflection of the LR_{UC} test and give a penalty of 1 to each violation of the VaR value. The BLF could be expressed as follows:

$$BLF_{t+1} = \begin{cases} 1 & \text{if } R_t < VaR_{t+1|t}(\alpha) \\ 0 & \text{if } R_t \geq VaR_{t+1|t}(\alpha) \end{cases}$$

An adequate VaR model should take in its account not only the count of violations but also the magnitude of it. The regulatory (or quadratic) loss function (RLF)[8] , shown below, includes a quadratic term to impose more penalty on big failures compared to the small failures (Sarama et al. 2003). A model is preferred to other candidate models if it yields a lower total loss value that is defined as the sum of these penalty scores $\sum RLF_{t+1}$ .The model that does not generate any violation is deemed the most adequate since the total loss function equals zero (Xekalaki and Degiannakis, 2010).

$$RLF_{t+1} = \begin{cases} \left(R_t - VaR_{t+1|t}(\alpha)\right)^{2} & \text{if } R_t < VaR_{t+1|t}(\alpha) \\ 0 & \text{if } R_t \geq VaR_{t+1|t}(\alpha) \end{cases}$$

3. Data and Empirical Results

3.1 Data and preliminary analysis

The data consists of 2520 daily observations of the MSCI-Turkey equity index from the period 2nd January 2006 to 28th August 2015 sourced from DataStream. This Index is designed to measure the performance of the large and mid-cap firms of the Turkish market. It covers about 85% of the equity universe in Turkey. Returns are constructed as the first difference of natural

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[8] The RLF is similar to the magnitude loss function of Lopez (1998)

logarithm of share prices times 100. Table 1 displays the descriptive statistics for the whole sample period. The mean return is insignificantly different from zero. Return series exhibits significant negative skewness and excess kurtosis. The Jarque –Bera (J-B) statistic confirms that the return series deviates from normality as the null hypothesis of unconditional normality is rejected beyond the 1% level of significance. To test the hypothesis of independence, Ljung-Box Q statistic for lag 10 is estimated for the returns and squared returns, $LB\ Q_{(10)}$ and $LB^2\ Q_{(10)}$ respectively; and also reported in Table 1. From these test statistics, the null of no serial correlation between returns could not be rejected, however, it has to be rejected for the squared returns. Given that the squared reruns are serially correlated, GARCH family models would be appropriate to capture the dynamics of variance. Before modelling return series using GARCH models, we test whether or it is a stationary process using the ADF (Dickey and Fuller, 1981) and KPSS (Kwiatkoski et.al, 1992) unit root tests. According to the ADF test, the null hypothesis that the return series contains a unit root has to be rejected at the traditional levels of significance. The KPSS confirms that conclusion since the null hypothesis that the return series is stationary cannot be rejected at conventional levels of significance. Accordingly, returns in their levels could be used in the following analysis[9].

3.2 Results of volatility models using the entire sample

Maximum likelihood estimates of the parameters of models under consideration are obtained by numerical maximization of the log-likelihood function, employing the Marquardt (1963) algorithm. Table 2 introduces the results of parameter estimates of the two models under the three employed conditional densities for the whole sample period. When adopting the Taylor GARCH(1,1) model to the entire dataset, it is found unjustified since convergence could not be achieved. For the two models under the three conditional densities, the mean equation that includes only an intercept is found sufficient to capture the dynamics of the mean equation as reflected by the insignificant Ljung-Box Q statistic up to lag 10, $LB\ Q_{(10)}$, for standardised residuals. The intercept is found to be significantly different from zero for GARCH(1,1) under the three distributions but insignificant under the APGARCH(1,1) model under the all employed densities.

Concerning the variance equation under the three conditional distributions, all coefficients are found to be significantly different from zero at the 1% level of significance. Since the asymmetry parameter γ_1 of APGARCH, under the three conditional densities, has a positive sign and is very significant, this indicates the presence of leverage effect implying a higher response to past negative shocks. The estimated asymmetry parameter is roughly four times higher than the estimated ARCH parameter. Thus, negative innovations at day t increase the volatility at day t+1 by around four times as a positive innovation of the same magnitude. The autoregressive effect in volatility is very strong. The GARCH parameter, β_1, ranges between a minimum of 0.8807 for the APGARCH(1,1)-GED model and a maximum of 0.9112 for the GARCH(1,1)-t model, suggesting that the volatility has a strong memory effect. The power term parameter δ equals 1.811, 1.829, and 1.796 for the APGARCH(1,1)-n, APGARCH(1,1)-t and APGARCH(1,1)-ged models respectively.

The estimated degrees of freedom parameters, v, of the conditional student t distribution which equal 6.663 and 6.890 for GARCH(1,1) and the APGARCH(1,1) models respectively is significant beyond 1% level of significance. Additionally, the tail thickness parameter, ξ, of GED distribution that equal 1.324 and 1.344 for GARCH(1,1) and the APGARCH(1,1) models respectively is also significant beyond 1% level of significance. This indicates that the

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[9] Results of the tests above are not reported due to limited space. However, they are available upon request from the authors.

standardized residuals are not normally distributed even after taking GARCH effects into consideration.

Model diagnostics [Ljung-Box Q statistic for lag 10 is estimated for the standardised residuals and squared standardized residuals, $LB\ Q_{(10)}$ and $LB^2\ Q_{(10)}$ respectively, and Lagrange Multiplier to test for the presence of ARCH effects in the residuals (LM-ARCH(10))] are presented in the bottom of Table 2. These tests indicate that standardized residuals and their squared counterparts from the employed models, irrespective of the conditional density used, are free from serial correlation up to lag 10 and that ARCH effects are not present in standardized residuals squared. Accordingly, both mean and variance equations are said to be well specified.

According to the log-likelihood statistic, APGARCH(1,1) model with GED innovations, followed by the APGARCH(1,1) model with Student t distribution, is considered the best model to fit the data. On the other hand, GARCH(1,1) with a normal distribution is ranked the last. This is not surprisingly since models with more parameters always maximize the likelihood function. Nevertheless, when the number of parameters is given consideration, as in the AIC and BIC, the conclusion above is still valid. However, the superiority of one model cannot be assessed by using goodness-of-fit tests. The GARCH (1,1) model under different densities is found to be stable since the joint Nyblom test statistic[10], joint L_C , is found less than critical one at 5% for all return series. However, for the APGARCH(1,1) model under the employed densities, the null parameter stability has to be rejected at 5% level of significance. Such conclusion justifies estimating volatility models using rolling window procedure.

Thus, the next subsection will deal with the model selection according to its out-of-sample criteria. In other words, models will be evaluated for their ability to generate the most accurate one-day-ahead VaR at 95% level of confidence. We test whether δ is significantly differ from unity (Taylor (1986) model) and two (Bollerselv (1986) model). According to the likelihood ratio statistics, the parameter δ is found to be significantly different from one but not from two[11]. Giot and Laurent (2003) found that δ is between 1.052 and 1.793 and mostly significantly different from 2. For five of the six series where δ is found insignificantly different from 1, they concluded that, instead of modelling the conditional variance (GARCH), it is more relevant to model the conditional standard deviation. The likelihood ratio test confirmed that the power transformation parameter for the entire data set is significantly different from unity. Hence, the Taylor GARCH(1,1) seems to be inapplicable to the daily MSCI-Turkey equity index data during the study period.

3.3 Results of volatility models using rolling window procedure

Using a constant rolling sample of 1800, we generate 720 one-step-ahead volatility forecasts using the above-observed series of the MSCI-Turkey equity index. We use the first 1800 observations to estimate the parameters of the two models under the three employed distributions. The estimation window is then moved forward by one day, and the model of concern is re-estimated. This process keeps running forward day by day until the end of the entire sample. The rolling estimation procedure allows for tracking the evolution of estimated parameters. Degiannakis et.al (2008) found that the estimated parameters of the EGARCH model change over time, however; the model does not lose its ability to forecast accurately one-day-ahead volatility. Xekalaki and Degiannakis (2010) asserted the time-varying parameters reflect the fact that structural properties and trading behaviour alter over time.

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[10] The joint L_C statistic tests the null that the entire vector of parameters is stable against the alternative that the entire vector may be unstable (i.e. following a martingale process). As argued by Nyblom (1989), this encompasses the case of one or more structural breaks. The test statistic has an asymptotic distribution which depends only on the number of estimated parameters. Nyblom (1989) and Hansen (1990) tabulated this distribution. It is worth mentioning that the test statistic is robust for heteroscedasticity.

[11] To save space, results are not shown. However, they are available upon request from the authors.

Figure 1 in Appendix depicts the rolling power transformation parameter under the three employed distributions12. Each figure plots the 95% confidence interval of each parameter. Confidence intervals are constructed employing the standard errors of the parameters estimated using the full data sample. Table 3 introduces the percentages of the rolling-sampled estimations which are outside the 95% confidence interval. The rolling estimates of power transformation parameter (δ) are outside the 95% confidence interval of the full-sampled (δ) in the 48.88%, 17.77%, and 23.19% of the cases under the normal, student t and GED distributions respectively. In many cases, we found that the rolled-over parameter of power transformation differ significantly from one and from two, implying that the volatility dynamics may switch from variance specification to another. From table (3), we conclude that 100% of degrees of freedom of Student t distribution and the tail thickness parameter of GED distribution are found to lie inside the 95% confidence interval. This implies that the leptokurtocity of MSCI-Turkey equity returns does not change over time during the study period. In contrast, Ané (2006) found that the degrees of freedom of the Student t change over time and, hence, the fat-tailedness of the distribution of the innovations to be time-variant. The reduction of kurtosis is approximately 30% for the employed Japanese indexes regardless the variance equation specification.

Table 4 introduces the mean and the standard deviation for the power transformation parameter δ obtained by the roll-over estimation procedure of a fixed window size of 1800 observations. Figure 2 displays the empirical distribution of the power transformation parameter under the three distributions. The densities are obtained by the kernel method. It is clear that the empirical distribution of the power transformation parameter δ shows clear bimodality. Such conclusion conforms to that of Ané(2006) regarding the Japanese financial stock data.

3.4 Implication for value-at-risk

To compare the alternative volatility models from a practical point of view, we use the one-day-ahead volatility forecasts to compute the one-step-ahead 5 percent VaR of a ₺[13]1000000 portfolio. Table 5 reports the results of out-of-sample VaR models using the 95% level of confidence. The percentage of negative returns smaller than the one-step-ahead VaR is not significantly differed from 5% for all estimated models. Thus, these models under different conditional densities pass the unconditional coverage test of correct failure rate. In addition, hits are found to be serially independent (i.e. they do not cluster). Hence, all models pass the conditional coverage test of correct failure rates and independence. Accordingly, all models are said to be statistically adequate in forecasting one-step-ahead VaR. Although the binary loss functions (BLF), expressed as the percentage of a number of hits, are never statistically different from 5%, the GARCH(1,1)-t and APGARCH(1,1)-t models are a little more conservative with a failure rate below the 5% target. According to the regulatory loss functions (RLF) taking into account not the number of failures but also their magnitudes, asymmetry, and fat tails seem to be important in modelling VaR since the APGARCH(1,1)-t is found to be the most efficient amongst its other competitors. The GARCH(1,1)-t is considered to be the second best in terms of efficiency. Hence, the conservative models, with conditional Student t density, seem to perform better than other competitors. On the other hand, the GARCH(1,1) with conditional normal innovations is considered to be the least efficient model in forecasting one-day-ahead VaR with the largest regulatory loss function.

The VaR statistics (mean, standard deviation, minimum, and maximum) in the table reflect losses in thousands of Turkish Lira on a portfolio of 1 million Turkish Lira. With an initial investment of ₺1 million, an investor loses, on average, about ₺ 29506.4, ₺29049.34, ₺25489.81, ₺25303.5, ₺24996.5, and ₺24688.12 when employing APGARCH(1,1)-t,

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[12] To save space, figures of other rolled parameters are not included. However, they are available upon request from the authors.

[13] ₺ stands for Turkish Lira

GARCH(1,1)-t, APGARCH(1,1)-GED, APGARCH(1,1)-n, GARCH(1,1)-GED, and GARCH(1,1)-n, respectively. The maximum amounts an investor loses are ₺ 81024.7, ₺68802.3, ₺72850.1, ₺74140.4, ₺60819.7, and ₺62214.2 using the models above respectively. As mentioned earlier, the APGARCH(1,1)-t, GARCH(1,1)-t are ranked the best and the second best in terms of quadratic loss function that takes into account both the number and size of hits. Panels A and B of Figure 1 plot the one-day-ahead VaR predictions and the realised returns, measured in terms of Turkish Lira, for GARCH and APGARCH models respectively. It is clear that VaR prediction generated by the employed models under different densities react to increase in turmoil periods and decrease in tranquil times. From Figure (1), with 95% confidence the APGARCH(1,1)-t, GARCH(1,1)-t predict that portfolio loss will be no more than ₺ 33134.29 and ₺ 28990 in 3rd, June 2013. However, the actual loss (the negative returns) was ₺107612.1. Thus, the magnitude of the hit to the predictions of these two models is ₺74477.81 and ₺78622.1 respectively. The GARCH(1,1) performs the worst with an exception of ₺82133.2. On 27th of February, 2015, the realized returns were higher than the VaR implied by all models except those with student t distribution [for example, the VaR was exceeded by ₺1332.4, ₺1064.4, and ₺7113.1 for normal-GARCH, normal-APGARCH, and GED-APGARCH, respectively]. Thus, it seems that volatility models with student t distribution introduce better hedge for market risk in the ISE.

4. Conclusion and Areas for Further Research

We investigate whether asymmetry and fat tails matter when modelling VaR for ISE. Thus, we employ the symmetric GARCH(1,1) model and Asymmetric Power GARCH(1,1) model, using normal distribution, the Student t, and the GED to account for leptokurtosis. Additionally, we examine if the estimated parameters are time-varying. We compare the forecasting performance of GARCH(1,1) and APGARCH(1,1) models using three different distributions, normal and Student t and GED, in the context of value at risk for ISE. We employ daily data of the MSCI-Turkey Equity Index for the period stretching from 2nd January 2006 to 28th August 2015. The two models under the three distribution were executed using the entire sample period, and the power transformation parameter was found significantly different from one but not from two. The employed models are evaluated through their ability to produce the most accurate one-day-ahead VaR at 95% level of confidence. Consequently, we used a rolling sample of constant size of 1800 to generate 720 one-step- ahead volatility forecasts to provide the one-step-head variance predictions required to create the one-step-ahead VaR. Employing the rolling window procedure to re-estimate the model parameters each trading day allowed to examine if the estimated parameters of the variance equation change over time. When we tested the stability of employed models, the joint Nyblom test statistic indicated the stability of GARCH(1,1) specifications under the different densities while the APGARCH (1,1) model is found unstable under the different distribution. This evidence was supported by running the rolling window estimation as the power transformation parameter (δ) are outside the 95% confidence interval of the full-sampled (δ) in the 48.88%, 17.77%, and 23.19% of the cases under the normal, student t and GED distributions respectively.

To check their statistical precision of the risk models, we first filtered it by back-testing measures of Kupiec (1995) and Christoffersen (1998). This is done to examine whether the mean number of violations is not statistically significantly different from that expected and whether these violations are independently distributed. The two models using the different three distributions passed the tests above of statistical adequacy. Then these adequate models are assessed using binary and loss function. Though the binary loss functions (BLF), are never statistically different from 5%, the GARCH(1,1)-t and APGARCH(1,1)-t models are a little more conservative with a failure rate below the 5% target. On the other hand, the regulatory loss functions that consider both the number of failures and their magnitudes, suggest that APGARCH(1,1)-t to be the most efficient amongst its other competitors. This implies that

asymmetry and fat tails are important in modelling the VaR. Further, The GARCH(1,1)-t is selected as the second best with respect to efficiency indicating that the conservative models, with conditional Student t density, are superior to other employed models. Finally, the GARCH(1,1) with normally distributed errors is regarded as the worst model in forecasting one-day-ahead VaR with the largest regulatory loss function. Given the superior of models with student t distribution, it seems that the distributional assumption plays a major role in forecasting volatility and predicting potential losses on the Istanbul Stock Exchange, and thus, they can be employed for a better hedge against market risk in the ISE.

Further areas of research include applying the methodology of Doornik and Ooms (2005) in detecting outliers in GARCH models since the presence of outliers affect forecasted volatility and, hence, the forecasted VaR. They recommend adopting their test since it can serve as a misspecification test. In addition, they argue that the detected outliers can complement value-at-risk estimations because, in large samples, the distribution of outliers is informative in itself. Given the integration of the ISE with other international fanatical markets (e.g. the U.S one), another area for further research would be estimating VaR for the ISE using multivariate GARCH models since the risk of one asset depends dynamically on its past risk as well as on the past risk of other assets (McAleer and da Veiga, 2008). Moreover, Thomas (2003) proposed a new multi-fractal model that account for long-memory effects in financial volatility. He found that comparing the multi-fractal forecasts with those derived from GARCH and FIGARCH models yields results in favour of the new model. Hence, a comparison of the performance of this new specification with other GARCH competitors in forecasting the VaR would be another area for further research.
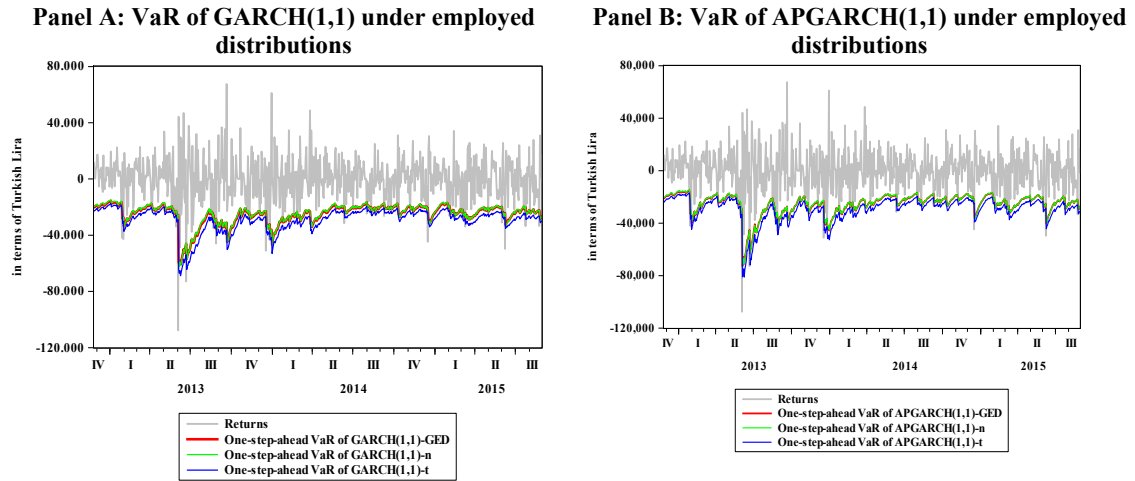
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Figure 1: MSCI-Turkey Returns and One-Step-Ahead VaR at 95% Level of Confidence

Panel A: VaR of GARCH(1,1) under employed distributions

Panel B: VaR of APGARCH(1,1) under employed distributions

Source: authors' calculations

Table 1: Descriptive Statistics of Daily Returns of MSCI-Turkey for the Whole Sample Period

No. of Obs.	Mean[1] t-statistic	Standard deviation	Skewness[2] t-statistic	Kurtosis[3] t-statistic	Minimum Maximum	J-B statistic [p-value]	LB $Q_{(10)}$ [p-value]	LB^2 $Q_{(10)}$ [p-value]
2519	0.0159 (1.752)	1.808	-0.142 (-2.909)*	6.485 (35.706)*	-10.761: 12.722	1263.24 [0.000]*	15.271 [0.122]	384.95 [0.000]*

Notes: (1) t-statistic, between parentheses, is calculated as t = mean return/(standard deviation * square root of the sample size). (2) t-statistic, between parentheses, is calculated as $t = (S' - 0) / SE(S')$, where S' is the value of skewness coefficient, 0 is the value of skewness coefficient for a normal distribution and $SE(S')$ is the standard error of the estimated skewness coefficient which calculated as the square root of 6/n, where n is the number of observations. (3) t-statistic, between parentheses, is calculated as $t = (K' - 3) / SE(K')$, where K' is the value of kurtosis coefficient, 3 is the value of kurtosis coefficient for a normal distribution, and $SE(K')$ is the standard error of the estimated kurtosis coefficient that calculated as the square root of 24/n, where n is the number of observations. (4) * indicate that the null hypothesis should be rejected at 1% level of significance or less.
Source: Authors' calculations.

Table 2: Estimates Results of Alternative Volatility Models: The Whole Sample Period

Parameters	GARCH(1,1)			APGARCH(1,1)		
	normal	Student t	GED	normal	Student t	GED
μ_0	0.0897 [0.002]	0.0795 [0.007]	0.0659 [0.020]	0.052 [0.08]	0.055 [0.062]	0.045 [0.111]
ω_0	0.0847 [0.001]	0.0618 [0.000]	0.0753 [0.000]	0.097 [0.000]	0.0924 [0.000]	0.0996 [0.000]
ω_1	0.0873 [0.000]	0.0709 [0.000]	0.0813 [0.000]	0.0813 [0.000]	0.0759 [0.000]	0.0831 [0.000]
γ_1	-	-	-	0.3415 [0.003]	0.3578 [0.0000]	0.3577 [0.000]
β_1	0.8880 [0.000]	0.9112 [0.000]	0.8996 [0.000]	0.8828 [0.000]	0.8897 [0.000]	0.8807 [0.000]
δ				1.811 [0.000]	1.829 [0.000]	1.796 [.000]
ν	-	6.633 [0.000]	-	-	6.890 [0.000]	-
ξ	-	-	1324 [0.000]	-	-	1.344 [0.000]
LB $Q_{(10)}$	12.315 [0.265]	12.802 [0.235]	12.584 [0.248]	10.676 [0.265]	10.702 [0.381]	10.650 [0.385]
LB^2 $Q_{(10)}$	5.411 [0.862]	7.782 [0.650]	5.829 [0.829]	4.873 [0.899]	5.055 [0.887]	5.247 [0.874]
LM-ARCH(10)	5.574 [0.849]	7.975 [0.631]	6.009 [0.814]	5.328 [0.868]	5.685 [0.840]	5.403 [0.862]
Joint L_C 5% critical value	0.7782 [1.24]	1.033 [1.47]	1.116 [1.47]	1.70 [1.68]	2.09 [1.9]	2.09 [1.9]
Log-Likelihood	-4871.367	-4817.781	-4815.033	-4851.54	-4804.768	-4801.118
AIC	3.870	3.829	3.826	3.856	3.820	3.817
BIC	3.880	3.840	3.838	3.870	3.836	3.823

Notes (1) p-values are in brackets below. (2) Coefficients in bold are significantly different from zero. (3) For models based on normal distribution, standard errors are calculated using the robust method of Bollerslev and Wooldridge (1992). (4) LB Q(10) and LB^2 Q(10) are the Box-Pierce portmanteau statistic for the first ten autocorrelations of standardised residuals and squared standardised residuals, respectively. (5) Joint L_C Indicates the test statistic of Nyblom(1989) and modified by Hansen (1990), which is asymptotically robust to heteroscedasticity, tests the null of the constancy of the entire vector of estimated parameter against an alternative that the entire vector may be unstable. The 5% critical values of the test statistic, which depends on upon the number of estimated parameters, are reported in square parentheses beneath the estimated test statistic. (6) AIC and BIC stand for the Akaike information criterion Bayesian information criterion.
Source: Authors' calculations

Table 3: Percentage of Rolling-Sampled Estimated Parameters That Lie Outside the 95% Confidence Interval

Parameter	GARCH(1,1)			APGARCH(1,1,1)		
	normal	Student t	GED	normal	Student t	GED
μ_0	**0**	0	0	**0**	0	0
ω_0	0	**0**	0	4.44%	**0**	0.55%
ω_1	0	**0.83%**	0	5.55%	**0**	1.80%
γ_1	-	-	-	2.63%	0	0
β_1	0	**0**	**0**	5.97%	**0**	**0.13**
δ	-	-	-	**48.88%**	**17.77%**	**23.19%**
ν	-	0	-	-	0	-
ξ	-	**-**	0	-	**-**	0

Source: Authors' calculations

Table 4: Statistics of the Rolling Power Transformation Parameter

	APGARCH(1,1)-n	APGARCH(1,1)-t	APGARCH(1,1)-GED
Mean	2.231	2.182	2.135
Standard deviation	0.335	0.391	0.368
Minimum	1.353	1.414	1.366
Maximum	3.205	3.193	3.144

Source: Authors' calculations.

Table 5: Testing the Models With Out-of-Sample VaR Statistics (95% level of confidence) VaRs in ₺

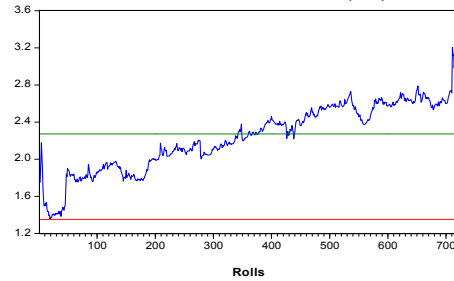|	GARCH (1,1)			APGARCH(1,1)		
	Normal	Student t	GED	Normal	Student t	GED
VaR mean	24688.12	29049.34	24996.5	25303.5	29506.4	25489.81
VaR std.	74436.6	8384.6	7337.5	8224.68	9332.1	8144.93
VaR max	62214.2	68802.3	60819.7	74140.4	81024.7	72850.1
VaR min	14833.9	1754.7	15404.6	14446	16915.5	14884.1
Expected Hits	36	36	36	36	36	36
Actual Hits	37	25	36	37	25	36
LR_{UC}	0.0289*	3.943**	0*	0.0289*	3.943**	0*
	[0.864]	[0.047]	[1]	[0.864]	[0.047]	[1]
LR_{CC}	0.596*	3.963*	0.465*	0.596*	3.963*	0.465*
	[0.742]	[0.137]	[0.792]	[0.742]	[0.137]	[0.792]
BLF%	0.0513	0.034	0.050	0.0513	0.034	0.050
BLF%	121.525	93.525	119.556	111.573	83.648	108.980
Rank	**6**	**2**	**5**	**4**	**1**	**3**

Notes: (1) p-values of test statistics are in brackets beneath them. (2) * indicates that the null has to be rejected at conventional levels of significance. (3) ** indicates that the null has to be rejected at 1% level of significance. (4) Models are ranked according to regulatory loss function (RLF) where the model with the lowest value is the best one. (5) VaR mean, VaR standard deviation (VaR std), maximum VaR (VaR max), and minmum VaR (VaR min) are expressed in terms of Turkish Lira for ₺ 1Million.
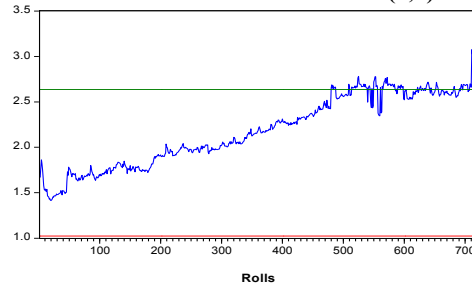Source: Authors' calculations

Appendix

Figure 1: The 720 estimates of the power transformation power parameters of the APGARCH(1,1,1) model on the basis of a rolling sample of 1800 observations on the MSCI-Turkey equity index
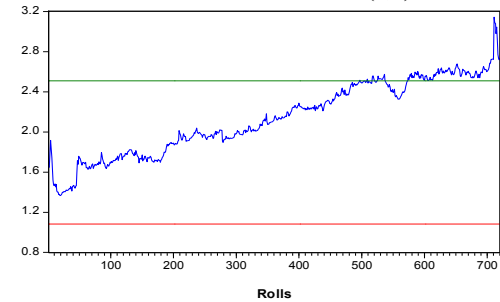
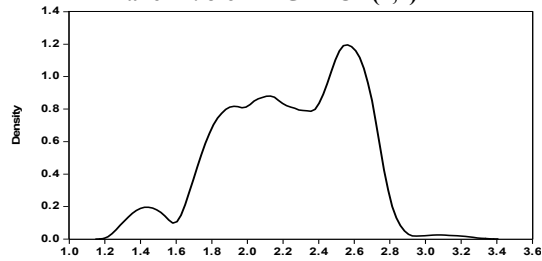**Panel A:$\delta^{(t)}$ under the APGARCH(1,1)-n model**　　**Panel B:$\delta^{(t)}$ under the the APGARCH(1,1)-t model**　　**Panel F:$\delta^{(t)}$ the APGARCH(1,1)-GED model**
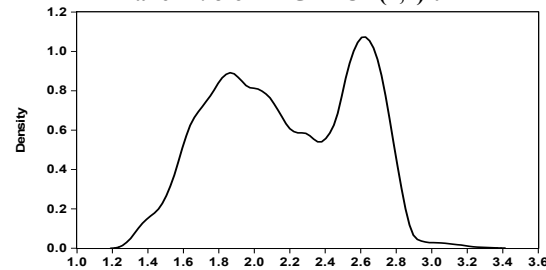

Source: Authors' calculations

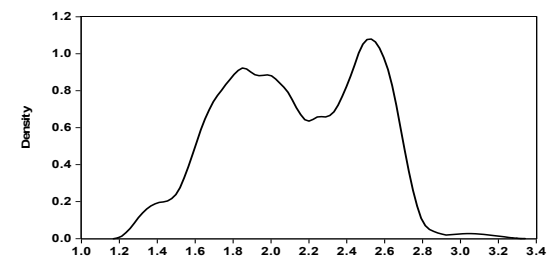**Figure 2: Empirical Distribution of The Power Transformation δ**

Panel A: δ of APGARCH(1,1)-n　　**Panel B: δ of APGARCH(1,1)-t**　　**Panel A: δ of APGARCH(1,1)-GED**

Source: Authors' calculation.